

The Possible $J^{PC}I^G = 2^{++}2^+$ State X(1600)

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The interesting state X(1600) with $J^{PC}I^G = 2^{++}2^+$ can't be a conventional $q\bar{q}$ meson in the quark model. Using a mixed interpolating current with different color configurations, we investigate the possible existence of X(1600) in the framework of QCD finite energy sum rules. Our results indicate that both the "hidden color" and coupled channel effects may be quite important in the multiquark system. We propose several reactions to look for this state.

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I. INTRODUCTION

There has been important progress in the experimental search of multiquark hadrons since last year. LEPS collaboration reported evidence of the Θ^+ pentaquark with $S = +1, B = +1$ and the minimum quark content $uudd\bar{s}$ [1]. Such a state is clearly beyond the conventional quark model if it is established by future experiments. A recent review of pentaquarks can be found in Ref. [2].

BABAR, CLEO and BELLE collaborations observed two narrow charm-strange mesons $D_{sJ}(2317), D_{sJ}(2457)$ below threshold [3]. These states are 160 MeV below quark model predictions. Some authors speculate they are four-quark states [4, 5]. These states may admit a small portion of DK or D^*K continuum contribution in their wave functions. But the dominant component of $D_{sJ}(2317), D_{sJ}(2457)$ should be $c\bar{s}$ [6]. This topic is reviewed in Ref. [7].

BELLE collaboration discovered a new narrow charmonium-like state X(3872) in the $J/\psi\pi^+\pi^-$ channel [8]. Its mass is very close to $D\bar{D}^*$ threshold. Its production rate is comparable to those of other excited charmonium states. Very recently BELLE collaboration observed the same signal in the $J/\psi\omega$ channel [9]. Its mass and decay pattern seems to favor its interpretation as a deuteron-like $D\bar{D}^*$ molecule [10]. Naively, one would expect the lower production rate for $D\bar{D}^*$ molecules.

This year SELEX collaboration reported a narrow state $D_{sJ}(2632)$ above threshold [11]. Its dominant decay mode is $D_s\eta$. Such an anomalous decay pattern strongly indicates $D_{sJ}(2632)$ is a four quark state in the $SU(3)_F$ **15** representation with the quark content $\frac{1}{2\sqrt{2}}(ds\bar{d} + s\bar{d}d + su\bar{u} + us\bar{u} - 2s\bar{s}\bar{s})\bar{c}$ [12]. Other possible interpretations were discussed in Refs. [13].

In the light meson sector, $f_0(980)/a_0(980)$ lies 10 MeV below the K^+K^- threshold. It's difficult to find a suitable position for them within the framework of quark model. So they were postulated to be candidates of kaon molecule or four quark states. Recently there has accumulated some evidence of the four-quark interpretation of the low lying scalar mesons from lattice QCD calculation [14, 15].

The study of multiquark states started nearly three decades ago in the MIT bag model [16, 17]. The reaction $\gamma\gamma \rightarrow \rho\rho$ was suggested in the search of $q\bar{q}q\bar{q}$ resonances. Later ARGUS collaboration found evidence of a four-quark state in the dominant partial wave $J^PJ_z = 2^+2$ in the reaction $\gamma\gamma \rightarrow \rho^0\rho^0 \rightarrow 4\pi$ [18]. Now the observed signal was named as $X(1600 \pm 100)$ with the quantum numbers $J^{PC}I^G = 2^{++}2^+$ [19].

In this work, we employ QCD finite energy sum rules (FESR) to explore whether there exists a resonance in the $J^{PC}I^G = 2^{++}2^+$ channel.

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II. FORMALISM

QCD sum rule approach has proven useful in extracting the masses of the ground state hadrons [20]. QSR approach can yield the absolute mass scale as the lattice QCD formalism. First one starts from the correlation function composed of the interpolating current which strongly couples to the hadron which one wants to study. The correlation function can be calculated using the operator product expansion (OPE) technique. As one approaches the resonance region from large Q^2 , the nonperturbative power corrections become important gradually. One gets the spectral density of the correlation function in terms of quark and gluon condensates at the quark level. The hadron mass enters the spectral density of the correlation function at the hadron level. With the quark hadron duality assumption, one can extract the hadron mass.

The construction of a suitable interpolating current is crucial. There are only two independent color structures for a tetraquark. Let's first focus on a pair of $q\bar{q}$. Since $\bar{3}_c \times 3_c = 1_c + 8_c$, there are only two ways to form a color singlet tetraquark from two pairs of $\bar{q}q$. Both pairs are either in the color-singlet or color-octet state simultaneously. We denote the corresponding interpolating currents as η^1, η^8 . If we focus on the two quarks, $3_c \times 3_c = \bar{3}_c + 6_c$. The anti-quark pair must be in either 3_c or $\bar{6}_c$ color state in order to get a color-singlet tetraquark. We denote the corresponding interpolating currents as η^3, η^6 . It's important to note that η^3, η^6 are linear combinations of η^1, η^8 . We refer the reader to the appendix for details.

We use a general interpolating current for X(1600) which is the linear combination of η^3 and η^6 .

$$\begin{aligned}\eta_{\mu\nu}(x) &= \eta_{\mu\nu}^{\bar{3}} + Y\eta_{\mu\nu}^6 \\ &= (Y+1)\bar{d}^l(x)\gamma_\mu u^l(x)\bar{d}^m(x)\gamma_\nu u^m(x) + (Y-1)\bar{d}^l(x)\gamma_\mu u^m(x)\bar{d}^m(x)\gamma_\nu u^l(x) + (g_{\mu\nu}\text{terms})\end{aligned}\quad (1)$$

where $Y = a + bi$ is a complex number. a and b are real numbers. The decay constant f_X for X(1600) is defined as

$$\langle 0|\eta_{\mu\nu}(0)|X(1600)\rangle = f_X \varepsilon_{\mu\nu} \quad (2)$$

where $\varepsilon_{\mu\nu}$ is the polarization tensor of X(1600) meson.

We consider the following correlation function

$$i \int d^4x e^{-ipx} \langle 0|T\{\eta_{\mu\nu}(x)\eta_{\alpha\beta}^+(0)\}|0\rangle = \Delta_{\mu\nu;\alpha\beta}(p)\Pi(p^2) + \dots \quad (3)$$

where

$$\Delta_{\mu\nu;\alpha\beta}(p) = \frac{1}{2} \left(\Delta_{\mu\alpha}(p)\Delta_{\nu\beta}(p) + \Delta_{\mu\beta}(p)\Delta_{\nu\alpha}(p) - \frac{2}{3}\Delta_{\mu\nu}(p)\Delta_{\alpha\beta}(p) \right), \quad (4)$$

$$\Delta_{\mu\nu}(p) = g_{\mu\nu} - p_\mu p_\nu / p^2. \quad (5)$$

In Eq. (3), we have kept the unique tensor structure for $J^{PC} = 2^{++}$ mesons: $\Delta_{\mu\nu;\alpha\beta}(p)$. We note in passing that the $q\bar{q}$ meson states with $J^{PC} = 2^{++}$ have been studied in [22, 23], where $L=1$ orbital excitation has to be introduced. For any of its four Lorentz indices, $\Delta_{\mu\nu;\alpha\beta}(p)$ satisfies:

$$q^\mu \Delta_{\mu\nu;\alpha\beta}(p) = 0 \quad (6)$$

$$\Delta_{\mu;\alpha\beta}^\mu(p) = 0. \quad (7)$$

The non-resonant $\rho^+\rho^+$ intermediate states do not contribute to this tensor structure.

The scalar complex function $\Pi(p^2)$ satisfies the following dispersion relation

$$\Pi(p^2) = \int \frac{\rho(s)}{s - p^2 - i\epsilon} ds \quad (8)$$

where $\rho(s)$ is the spectral density. For a narrow resonance

$$\rho(s) = f_X^2 \delta(s - M_X^2) + \text{higher states}. \quad (9)$$

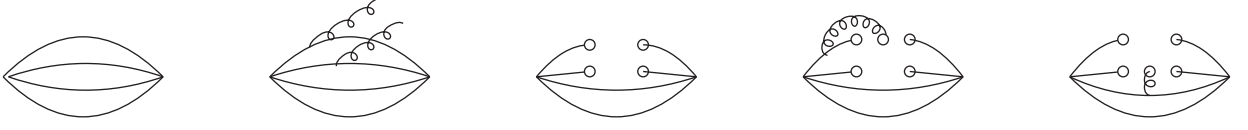


FIG. 1: The Feynman diagrams in the calculation of the two-point correlation function in Eq. (3).

At the quark gluon level the correlation function (3) reads

$$\begin{aligned}
i \int d^4x e^{-ipx} \{ & |Y+1|^2 [\text{Tr}[\gamma_\mu S^{jn}(x) \gamma_\alpha S^{mi}(-x)] \times \text{Tr}[\gamma_\nu S^{im}(x) \gamma_\beta S^{nj}(-x)] \\
& - \text{Tr}[\gamma_\mu S^{jn}(x) \gamma_\alpha S^{mj}(-x) \gamma_\nu S^{im}(x) \gamma_\beta S^{mi}(-x)] \\
& + 2(|Y|^2 - 1) [\text{Tr}[\gamma_\mu S^{jm}(x) \gamma_\alpha S^{mi}(-x)] \times \text{Tr}[\gamma_\nu S^{in}(x) \gamma_\beta S^{nj}(-x)] \\
& - \text{Tr}[\gamma_\mu S^{jm}(x) \gamma_\alpha S^{mj}(-x) \gamma_\nu S^{in}(x) \gamma_\beta S^{mi}(-x)] \\
& + |Y-1|^2 [\text{Tr}[\gamma_\mu S(x) \gamma_\alpha S(-x)] \times \text{Tr}[\gamma_\nu S(x) \gamma_\beta S(-x)] \\
& - \text{Tr}[\gamma_\mu S(x) \gamma_\alpha S(-x) \gamma_\nu S(x) \gamma_\beta S(-x)] + (\alpha \leftrightarrow \beta) \}
\end{aligned} \quad (10)$$

where $iS(x) = \langle 0|T\{q(x)\bar{q}(0)\}|0\rangle$ is the full quark propagator in the coordinate space. The Tr denotes the summation of both the color and Lorentz indices. Throughout our calculation, we assume the up and down quarks are massless. The first few terms of quark propagator is

$$iS^{ab}(x) = \frac{i\delta^{ab}}{2\pi^2 x^4} \hat{x} + \frac{i}{32\pi^2} \frac{\lambda_{ab}^n}{2} g_c G_{\mu\nu}^n \frac{1}{x^2} (\sigma^{\mu\nu} \hat{x} + \hat{x} \sigma^{\mu\nu}) - \frac{\delta^{ab}}{12} \langle \bar{q}q \rangle + \frac{\delta^{ab} x^2}{192} \langle g_s \bar{q} \sigma G q \rangle + \dots \quad (11)$$

The relevant terms that contribute to this correlator are represented pictorially in Figure 1.

After making Fourier transformation to $\Pi(x)$ we arrive at $\Pi(p^2)$. From the imaginary part of $\Pi(p^2)$ we extract the spectral density $\rho(s)$.

$$\begin{aligned}
\rho(s) = & \frac{1}{2^{12} \cdot 7 \cdot \pi^6} (1 + \frac{2}{3} |Y|^2) s^4 - \frac{1}{2^{14} \cdot 15 \cdot \pi^6} (23|Y|^2 - 21(Y + Y^*) + 47) s^2 \langle g_s^2 GG \rangle \\
& + \frac{5}{18 \cdot \pi^2} (1 - \frac{2}{5} |Y|^2) s \langle \bar{q}q \rangle^2 - \frac{1}{144 \cdot \pi^2} (13|Y|^2 - 3(Y + Y^*) - 33) \langle \bar{q}q \rangle \langle g_s \bar{q} \sigma G q \rangle
\end{aligned} \quad (12)$$

where we have used the factorization approximation for the high-dimension quark condensates.

III. FESR AND NUMERICAL ANALYSIS

In the Borel sum rule (BSR) analysis there are two parameters: the continuum threshold s_0 and Borel mass M_B . FESR contains a single parameter s_0 [24]. The dimension of the interpolating currents of the conventional hadrons like rho and nucleon is not high. Both BSR and FESR yield roughly the same results. If the interpolating current is of high dimension, the working window of M_B does not exist sometimes. In this case, FESR may have some advantage over BSR [25].

With the spectral density, the n th moment of FESR is defined as

$$W(n, s_0) = \int_0^{s_0} ds s^n \rho(s) \quad (13)$$

where $n \geq 0$. With the quark hadron duality assumption we get the finite energy sum rule

$$W(n, s_0)|_{Hadron} = W(n, s_0)|_{QCD} . \quad (14)$$

The mass can be obtained as

$$M^2 = \frac{W(n+1, s_0)}{W(n, s_0)}. \quad (15)$$

In principle, one can extract the threshold self-consistently from the requirement that the hadron mass has the least dependence on s_0 , i.e., $\frac{dM^2}{ds_0} = 0$. However, the threshold value s_0 extracted this way is not necessarily physical sometimes. The weight function of FESR enhances the continuum part even more than the weight function e^{-s/M_B^2} in the Borel sum rule. One must make sure that only the lowest pole contributes to the FESR below s_0 . Otherwise the result will be very misleading. To be more specific, a *naive* stability region in s_0 is no guarantee of a *physically reasonable* value for s_0 . For example, the FESR with an extracted threshold $s_0 \approx 10 \text{ GeV}^2$ is certainly irrelevant for the possible $X(1600)$ state.

Besides the weak dependence on s_0 , we also require (1) the zeroth moment $W(0, s_0) > 0$ and (2) the convergence of the operator product expansion (OPE). Higher dimension condensates should be suppressed in order to ensure a reliable FESR based on the converging OPE series. This convergence requirement is not easily satisfied if the interpolating current is of high dimension. As shown in the appendix, neither η^3 nor η^6 leads to a converging FESR for a reasonable value of s_0 . Only with a mixed current which is the linear combination of η^3 and η^6 , can we make the OPE series of FESR converging.

We use the following values of condensates: $\langle \bar{q}q \rangle = -(0.24 \text{ GeV})^3$, $\langle g_s^2 GG \rangle = (0.48 \pm 0.14) \text{ GeV}^4$, $\langle g_s \bar{q}\sigma Gq \rangle = -m_0^2 \times \langle \bar{q}q \rangle$, $m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$. To simply numerical analysis, we introduce variables ϵ_1, ϵ_2 which are defined as:

$$\begin{aligned} a^2 + b^2 &= \frac{5}{2}(1 - \epsilon_1) \\ a &= -\frac{1}{12}(1 + \epsilon_2). \end{aligned}$$

They satisfy two constraints: $\epsilon_1 \leq 1, |1 + \epsilon_2| \leq 6\sqrt{10(1 - \epsilon_1)}$. Now the spectral density reads

$$\begin{aligned} \rho(s) &= \frac{1}{2^{12} \cdot 21 \cdot \pi^6} (8 - 5\epsilon_1) s^4 - \frac{1}{2^{15} \cdot 15 \cdot \pi^6} (216 - 115\epsilon_1 + 7\epsilon_2 + 7) s^2 \langle g_s^2 GG \rangle \\ &\quad + \frac{5}{18 \cdot \pi^2} \epsilon_1 s \langle \bar{q}q \rangle^2 + \frac{1}{288 \cdot \pi^2} (65\epsilon_1 - \epsilon_2) \langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle. \end{aligned} \quad (16)$$

From Eq. (16) it's clear that we can adjust the variables ϵ_1, ϵ_2 to suppress $D = 6, 8$ condensates. In fact there exists a parameter space of (ϵ_1, ϵ_2) , with which there exists a stable FESR plateau in the M_X vs s_0 curve. The extracted s_0 at the stable plateau is physically reasonable. With this s_0 the zeroth FESR moment is positive, and the OPE is convergent. One typical set of these variables is $\epsilon_1 = -0.02, \epsilon_2 = -2$. After we divide each piece in the FESR by the perturbative term, the zeroth moment reads

$$W(0, s_0) \sim 1 - \frac{3.53}{s_0^2} - \frac{2.79}{s_0^3} - \frac{1.95}{s_0^4}. \quad (17)$$

The positivity requirement leads to $s_0 > 2.3 \text{ GeV}^2$.

For $s_0 > 2.3 \text{ GeV}^2$, the OPE also converges well. The variation of M_X with s_0 is presented in Figure 2. The extracted mass is

$$M_X = (1.65 \pm 0.15) \text{ GeV}. \quad (18)$$

As can be seen from Figure 2, the value of M_X does not change much if we truncate the OPE at $D = 6$. This is another sign of convergence of our FESR.

IV. DISCUSSION

The state $X(1600)$ with $J^{PC}I^G = 2^{++}2^+$ has inspired many theoretical papers [26]. The potential scattering of $\rho^0 \rho^0$ via the σ meson exchange was proposed to explain the signal in Ref. [27]. In the diquark cluster model [28], the mass and decay width of $X(1600)$ is studied together with other tetraquarks assuming the $\bar{\mathbf{3}}_c$ color wave function for two quarks. Using the potential model $X(1600)$ mass was estimated to be 1544 MeV recently [29].

In this work we have investigated this state using QCD finite energy sum rule. With any single current $\eta^1, \eta^8, \eta^{\bar{3}}, \eta^6$, the operator product expansion of the corresponding correlator does not converge. After we consider the linear

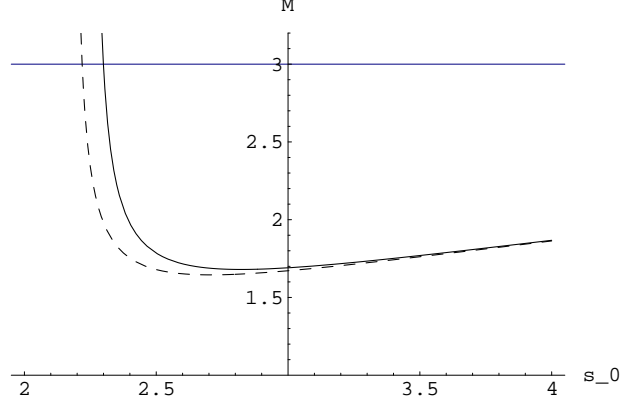


FIG. 2: The variation of the mass M with the threshold s_0 (in unit of GeV^2) for the current (1). The long-dashed and solid curves correspond to FESR when truncated at dimension $D = 6$ and $D = 8$ respectively.

combination of η^3 and η^6 , we have derived a converging FESR and observe a resonance signal at 1.65 GeV, which is close to the experimentally observed $X(1600)$ state. Our analysis indicates both the "hidden color" and coupled channel effects may be important in the multiquark system.

From charge, angular momentum, parity, isospin, C parity, and G parity conservation, we may obtain the allowed decay modes of $X(1600)$. G parity requires even number of pions in the final state for pure pion final states. For example, $X^0(1600) \rightarrow \rho^0 \pi^0, \omega \pi^+ \pi^-$ is forbidden by G parity. C parity forbids $X^0(1600) \rightarrow \omega(3\pi^0)$. Isospin conservation forbids the following decay modes: $X^0(1600) \rightarrow \omega \pi^0, \omega \omega$. The possible modes are $X^0(1600) \rightarrow \pi^0 \pi^0, \pi^+ \pi^-, 4\pi, \rho^+ \rho^-, \rho^0 \rho^0$. Angular momentum conservation requires D-wave decay for the two pion mode. Therefore, two pion decay modes may be suppressed compared with S-wave $\rho\rho$ mode.

Filippi et al. reported the possible existence of the $J^{PC} I^G = 0^{++} 2^+$ state decaying into $\pi^+ \pi^+$ in the reaction $\bar{n} p \rightarrow \pi^+ \pi^+ \pi^-$ [30]. On the other hand, some constraint has been obtained on the possible resonance in the scalar isotensor channel from the phase shift analysis of $\pi\pi \rightarrow \rho\rho \rightarrow \pi\pi$ scattering [31]. Constraint on the D-wave $I=2$ resonance from the similar analysis of the same reactions will be very desirable.

The process $\gamma\gamma \rightarrow \rho\rho \rightarrow X(1600) \rightarrow 4\pi$ was suggested and used in the search of $X(1600)$. We suggest the following reactions to look for this charming state.

- J/ψ decays at BES, CLEO and Υ decays at BELLE, BABAR

Symmetry considerations requires $X(1600)$ is produced together with odd number of pions in J/ψ decays. One pion mode is forbidden by isospin conservation. So the favorable decay chain is $J/\psi \rightarrow X(1600) + 3\pi \rightarrow 2\rho + 3\pi \rightarrow 7\pi$. There is also some chance in $J/\psi \rightarrow X(1600) + 3\pi \rightarrow 2\pi + 3\pi \rightarrow 5\pi$.

- Hadron reactions especially charge exchange processes

Charge exchange processes are useful for the production of $X^{++}(1600)$ if such a state exists. Some of them are $\pi^+ N \rightarrow \pi^- + X^{++}(1600) + N \rightarrow 5\pi + N$, $p + N \rightarrow n + N' + X^{++}(1600)$ where N is either a nucleon or nuclei. The later process can be studied at CSR facility at Lan Zhou.

- Anti-proton annihilation on the proton or deuteron targets

Let's take $X(1600)$ production from anti-proton annihilation on the proton target as an example. Since the isospin of $p\bar{p}$ is either 0 or 1, $X(1600)$ should be accompanied by one or several pions. However, kinematics allows only one pion decay mode: $\bar{p} + p \rightarrow X(1600) + \pi^0 \rightarrow 5\pi$. So we get $C_{p\bar{p}} = (-)^{L_{p\bar{p}} + S_{p\bar{p}}} = +, I_{p\bar{p}} = 1$.

(1) When the $p\bar{p}$ pair is in the S-wave, $L_{p\bar{p}} = S_{p\bar{p}} = J_{p\bar{p}} = 0, P_{p\bar{p}} = -$. Angular momentum and parity conservation requires $L_{\pi X(1600)} = 2$; (2) When the $p\bar{p}$ pair is in the P-wave, $L_{p\bar{p}} = S_{p\bar{p}} = 1, P_{p\bar{p}} = +, J_{p\bar{p}} = 1$ or 2. We get $L_{\pi X(1600)} = 1$; (3) When the $p\bar{p}$ pair is in the D-wave, $L_{p\bar{p}} = 2, S_{p\bar{p}} = 0, J_{p\bar{p}} = 2, P_{p\bar{p}} = -$. We have $L_{\pi X(1600)} = 0$.

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V. APPENDIX

In this appendix we list the currents with different color structure and their spectral densities. The color-singlet $\rho^+(x)\rho^+(x)$ type interpolating current reads

$$\eta_{\mu\nu}^1(x) = \bar{d}^i(x)\gamma_\mu u^i(x)\bar{d}^j(x)\gamma_\nu u^j(x) - 1/4 g_{\mu\nu}\bar{d}^i(x)\gamma_\rho u^i(x)\bar{d}^j(x)\gamma^\rho u^j(x), \quad (19)$$

which "i, j" are the color indices.

The current with color octet structure reads

$$\eta_{\mu\nu}^8(x) = \bar{d}^i(x)\left(\frac{\lambda^a}{2}\right)_{ij}\gamma_\mu u^j(x)\bar{d}^m(x)\left(\frac{\lambda^a}{2}\right)_{mn}\gamma_\nu u^n(x) - \frac{1}{4}g_{\mu\nu}\bar{d}^i(x)\left(\frac{\lambda^a}{2}\right)_{ij}\gamma_\rho u^j(x)\bar{d}^m(x)\left(\frac{\lambda^a}{2}\right)_{mn}\gamma^\rho u^n(x) \quad (20)$$

where $\frac{\lambda^a}{2}$ is the $SU(3)_c$ generator.

Using the identity of the λ matrix

$$\sum_a \frac{\lambda_{ij}^a}{2} \cdot \frac{\lambda_{kl}^a}{2} = \frac{1}{2}(\delta_{il}\delta_{jk} - \frac{1}{3}\delta_{ij}\delta_{kl}) \quad (21)$$

we can rewrite Eq. (20) as

$$\eta_{\mu\nu}^8(x) = -\frac{1}{6}\bar{d}^l(x)\gamma_\mu u^l(x)\bar{d}^m(x)\gamma_\nu u^m(x) + \frac{1}{2}\bar{d}^l(x)\gamma_\mu u^m(x)\bar{d}^m(x)\gamma_\nu u^l(x) + (g_{\mu\nu}\text{terms}) . \quad (22)$$

Alternatively, when two quarks are in the $\bar{\mathbf{3}}_c$ state, we have

$$\eta_{\mu\nu}^{\bar{\mathbf{3}}_c}(x) = \bar{d}^l(x)\gamma_\mu u^l(x)\bar{d}^m(x)\gamma_\nu u^m(x) - \bar{d}^l(x)\gamma_\mu u^m(x)\bar{d}^m(x)\gamma_\nu u^l(x) + (g_{\mu\nu}\text{terms}) .$$

The current with the $\mathbf{6}_c$ color structure is

$$\eta_{\mu\nu}^{\mathbf{6}_c}(x) = \bar{d}^l(x)\gamma_\mu u^l(x)\bar{d}^m(x)\gamma_\nu u^m(x) + \bar{d}^l(x)\gamma_\mu u^m(x)\bar{d}^m(x)\gamma_\nu u^l(x) + (g_{\mu\nu}\text{terms}) .$$

The spectral densities of the above four currents

$$\rho^1(s) = \frac{5}{2^{14} \cdot 21 \cdot \pi^6} s^4 - \frac{7}{2^{12} \cdot 15 \cdot \pi^6} s^2 \langle g_s^2 GG \rangle + \frac{1}{24\pi^2} s \langle \bar{q}q \rangle^2 + \frac{7}{288 \cdot \pi^2} \langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle . \quad (23)$$

$$\rho^8(s) = \frac{1}{2^{13} \cdot 27 \cdot \pi^6} s^4 - \frac{127}{2^{16} \cdot 135 \cdot \pi^6} s^2 \langle g_s^2 GG \rangle + \frac{1}{36 \cdot \pi^2} s \langle \bar{q}q \rangle^2 + \frac{131}{64 \cdot 81 \cdot \pi^2} \langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle . \quad (24)$$

$$\rho^{\bar{\mathbf{3}}_c}(s) = \frac{1}{2^{12} \cdot 7 \cdot \pi^6} s^4 - \frac{47}{2^{14} \cdot 15 \cdot \pi^6} s^2 \langle g_s^2 GG \rangle + \frac{5}{18 \cdot \pi^2} s \langle \bar{q}q \rangle^2 + \frac{11}{48 \cdot \pi^2} \langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle . \quad (25)$$

$$\rho^{\mathbf{6}_c}(s) = \frac{1}{2^{11} \cdot 21 \cdot \pi^6} s^4 - \frac{23}{2^{14} \cdot 15 \cdot \pi^6} s^2 \langle g_s^2 GG \rangle - \frac{1}{9 \cdot \pi^2} s \langle \bar{q}q \rangle^2 - \frac{13}{144 \cdot \pi^2} \langle \bar{q}q \rangle \langle g_s \bar{q}\sigma Gq \rangle . \quad (26)$$

In order to study the convergence of OPE series in FESR, we divide each piece in the zeroth moments by the four quark condensate.

$$\begin{aligned} W^1(0, s_0) &\sim 7.5 \times 10^{-3} s_0^3 - 4.7 \times 10^{-2} s_0 + 1 - \frac{0.9}{s_0} . \\ W^8(0, s_0) &\sim 3.5 \times 10^{-3} s_0^3 + 8.9 \times 10^{-3} s_0 + 1 - \frac{1.5}{s_0} . \\ W^{\bar{\mathbf{3}}_c}(0, s_0) &\sim 2.7 \times 10^{-3} s_0^3 + 1.2 \times 10^{-2} s_0 + 1 - \frac{1.3}{s_0} . \\ W^{\mathbf{6}_c}(0, s_0) &\sim -9.0 \times 10^{-3} s_0^3 + 2.9 \times 10^{-2} s_0 + 1 - \frac{2.6}{s_0} . \end{aligned} \quad (27)$$

Clearly these moments converge only when the threshold parameter is very large, $s_0 > 10 \text{ GeV}^2$. Such a large s_0 is irrelevant for $X(1600)$.

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